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Short Communication

Primitives Used in the Spatial Localization of Nonabutting Stimuli: Peaks or Centroids

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In order to determine whether simple luminance profiles are located by their peaks or centroids we performed a three element alignment task where the central element's degree of luminance asymmetry was randomly chosen from a flat distribution (skew noise). The central element with its randomly chosen skew was either positioned using the peak or centroid of its distribution. Accuracy is invariant with the magnitude of the skew noise for the centroid but not the peak condition. We conclude that the human visual system assigns position tags using centroids not peaks of luminance distributions for gabors. However this is not the case for Gaussian blobs, where a measure closer to the midpoint is used for our stimulus arrangement. Copyright © 1996 Published by Elsevier Science Ltd.

INTRODUCTION

A number of recent studies have suggested that stimuli that are well localized in space and frequency (i.e. gabors) are subjected to nonlinear filtering before localization [Toet & Koenderink (1988); Hess & Holliday (1992); Levi & Klein (1992); but also see Kooi et al. (1991)]. This nonlinear filtering extracts the contrast energy [see Hess & Holliday (1992) for one such model] and thus the stimuli are localized by virtue of their contrast envelopes. This appears to be true both for the fovea and periphery (Hess & Hayes, 1994).

There is, however, no general consensus on what primitive is used for localization beyond the filtering stage for nonabutting stimuli. A number of possibilities exist: centroids, edges, peaks, etc. Although edges are an unlikely candidate for stimuli with Gaussian profiles (for example, randomizing the Gaussian σ does not disrupt performance; Keeble and Hess, unpublished), centroids and peaks offer plausible alternatives. The majority of previous studies which have concentrated on alignment performance for abutting stimuli for which so called "hyperacuity" performance is obtained, support the centroid-alignment rule [among others, Westheimer & McKee (1977); Badcock & Westheimer (1985); Morgan & Aiba (1985); Watt & Morgan (1983)]. However Hess and Holliday (1992) found that the results of well separated Gaussian-weighted luminance distributions (e.g. gabors and Gaussian blobs) for which hyperacuity performance is never attained, were better modeled using a peaks-align rule. More recently, McGraw *et al.* (1995) and Whitaker *et al.* (*pers. comm.*) using a similar alignment task for well separated stimuli, but asymmetric Gaussians, found that a centroid rule best described their results.

It is a distinct possibility that the visual system can, via higher level cognitive processes, avail itself of a number of different cues depending on the perceived shape and/or distribution of the stimuli to be localized (Toet, 1987, 1988; Hess & Holliday, 1992; Hess *et al.*, 1994; Badcock *et al.*, 1996). For example, one could intellectualize and compensate in one's alignment by a factor which depends on the perceived skew of the stimulus. If this is true it might be argued that it is hardly surprising that centroids are used to align asymmetric stimuli and peaks to align symmetric stimuli. In order to make it impossible for higher cognitive processes to determine the nature of the primitive used based on the stimulus's luminance profile, we used a variant of the approach used by McGraw *et al.* (1995) in which:

1. Skewed luminance distributions were positioned with respect to symmetric reference stimuli using either their peaks or their centroids; and
2. The degree of skew in the luminance profile randomly varied from trial to trial.

In this way we introduced various amounts of peak-based noise when the central stimulus was positioned using its centroid and various amounts of centroid-based

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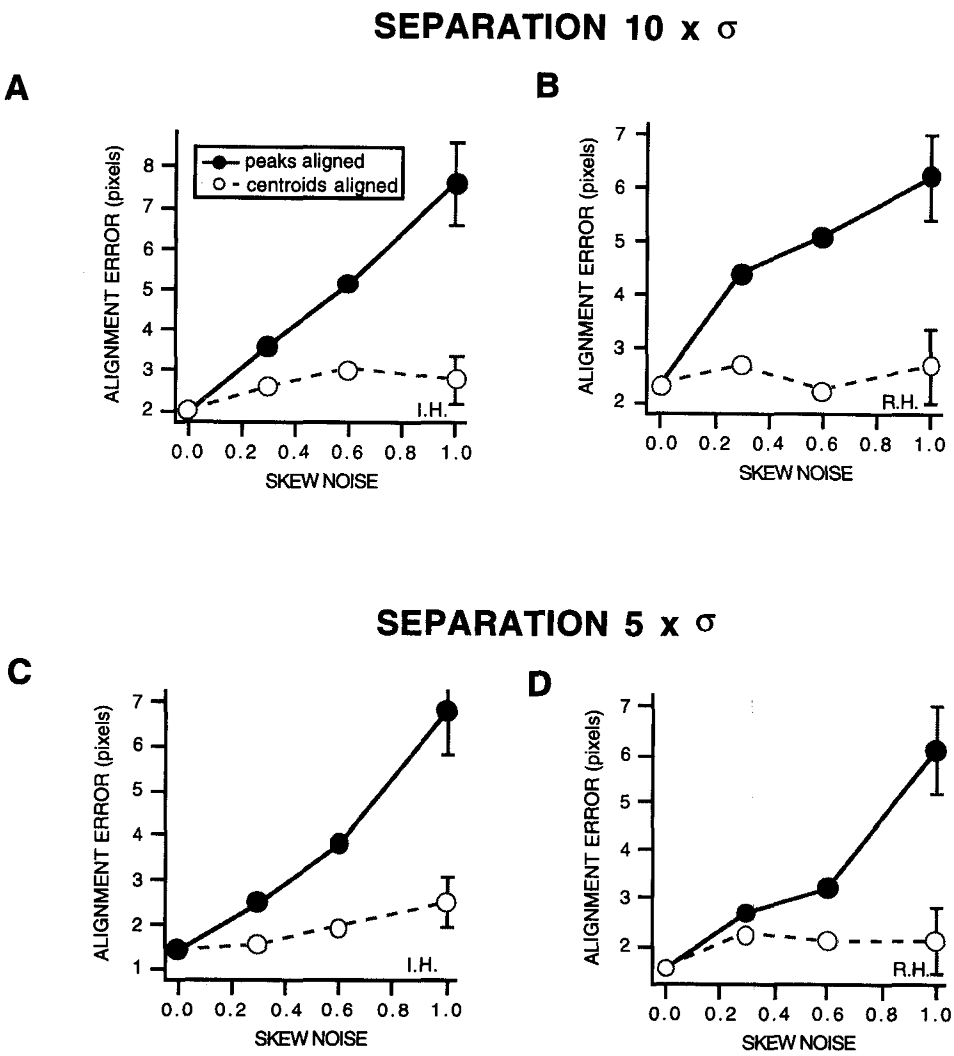

FIGURE 1. Alignment error in pixels is plotted against the magnitude of the skew noise for the central stimulus. The stimuli are Gaussian luminance profiles. Results are displayed for two subjects and for two reference element separations. The central asymmetric Gaussian was positioned either by its peak or its centroid. The fact that performance deteriorates as a function of skew noise when the central stimulus is positioned by its peaks and not when it is positioned by its centroid suggests that it is the centroid which is used to locate the stimulus. The error bars represent ± 1 SD.

noise when the stimulus was positioned using its peak. This confounds the use of alternate cues based on cognitive factors because the testing grid on which the central elements were positioned was uncorrelated with the degree or polarity of skew of the stimulus's luminance profile. Not only does the degree of skew vary randomly but also for the contrast level and presentation time used, neither the degree of skew nor its polarity was perceived except for stimuli at the extreme ends of the distribution. If the visual system uses either peaks or centroids then performance should be invariant with the magnitude of the skew noise when the stimuli are positioned about these respective primitives. If on the other hand, only one primitive is used then this task will identify whether it is the peak, the centroid or some other feature of the luminance distribution.

The results suggest that under conditions where higher cognitive factors are restricted, centroids and not peaks are used to locate the position of gabors. Gaussian blobs are located by a measure closer to the midpoint.

METHODS

Apparatus

All the stimuli were presented on a Joyce Electronics display screen with a P4 phosphor. The display was refreshed at 99 Hz, and had a vertical 100 kHz raster. The dimensions of the display area were 30×20 cm. The mean luminance of the display was 300 cd/m^2 [see Hess & Holliday (1992) for more details].

Stimuli

The stimuli were luminance patches or patches of sinusoidal grating enveloped in both the x- and y-dimensions by a Gaussian-based envelope. The orientation of the grating component of the stimuli was vertical.

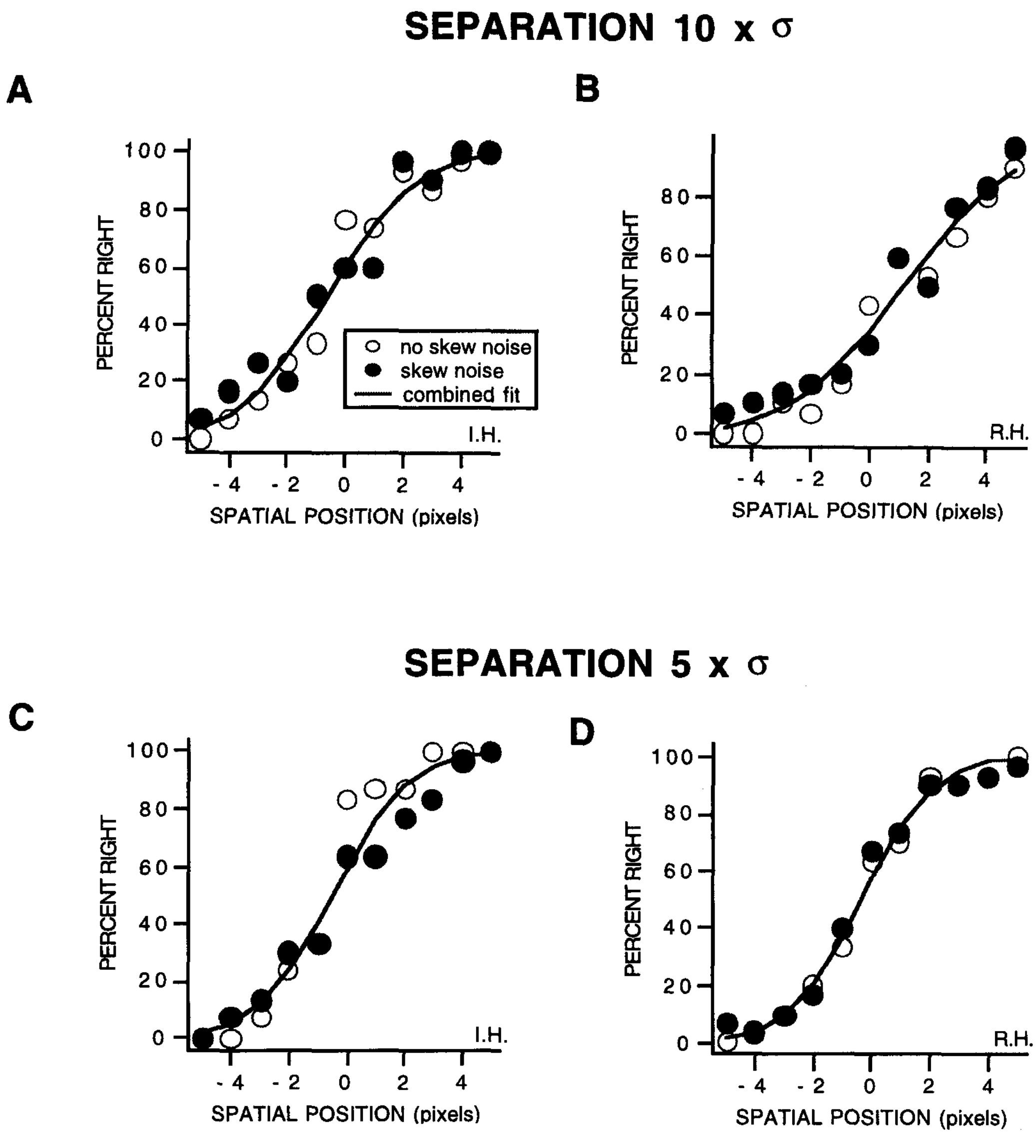

FIGURE 2. Psychometric data for the centroid-position condition are compared for the case of no skew noise (open symbols) and maximal skew noise (filled symbols). The stimuli are Gaussian luminance profiles. Results are shown for two subjects and two reference separations. The solid curve is the best fitting solution to the combined data for Eq. (2). See Table 1 for statistics.

The envelope was symmetric about the horizontal x-axis but not about the vertical y-axis. The form of the gabor-like function was:

$$G(x,y) = A * \cos(2\pi\omega(X - x)) * \exp(-\frac{1}{2}((X - x^2)/(\sigma_x^2) + (Y - y)^2/(\sigma_y^2))) \quad (1)$$

where A is the amplitude of the function, ω is the spatial frequency of the sinusoid, and σ_y is the standard deviation of the Gaussian envelope defining the patch in the y-direction. The Gaussian σ_y = 5.6 min. The horizontal σ_x was σ_L for x values $<X$ and σ_R for x values $>X$. The Michaelson contrast was set to 30% for the Gaussian luminance blobs and to 50% for the gabor-like stimuli.

Alignment

We measured the accuracy and point of subjective equality for the alignment of a central stimulus which could be judged relative to two vertically aligned reference stimuli [see Hess & Holliday (1992) for more details]. From the resulting psychometric function performance measures were derived by fitting the error function, ERF (x), of the form;

$$P(x) = A * (0.5 + 0.5 * \mathrm{ERF}((X - B)/(\sqrt{2}) * C))) \quad (2)$$

where A is the number of presentations per stimulus condition, B is the offset of the function relative to zero or the point of subjective equality measure, and C is the slope parameter of the function, which corresponds to the standard deviation of the assumed underlying normal distribution. The slope is the measure of the alignment error and the point of subjective equality is the measure of the centering of the function (50% point).

Skew noise

On each trial, a different central stimulus was chosen

TABLE 1. Maximum likelihood fits to the interlaced psychometric data for the no skew noise vs maximum skew noise conditions

Exp. condition	Model	χ^2	P of model fit
I.H. sep 10 × σ	separate μ & σ	8.4	0.97
	same μ & σ	9.1	0.95
I.H. sep 5 × σ	separate μ & σ	6.0	0.99
	same μ & σ	11.4	0.88
R.F.H. sep 10 × σ	separate μ & σ	7.6	0.98
	same μ & σ	9.7	0.93
R.F.H. sep 5 × σ	separate μ & σ	4.6	0.99
	same μ & σ	5.0	0.99
R.F.H. SF = 10 c/deg	separate μ & σ	5.2	0.99
	same μ & σ	5.3	0.99
R.F.H. sf = 5 c/deg	separate μ & σ	3.6	0.99
	same μ & σ	3.7	0.99

Two models were evaluated, one which assumes that the underlying data have the same μ & σ and another which assumes that they have different μ & σ. A single model can account for this data.

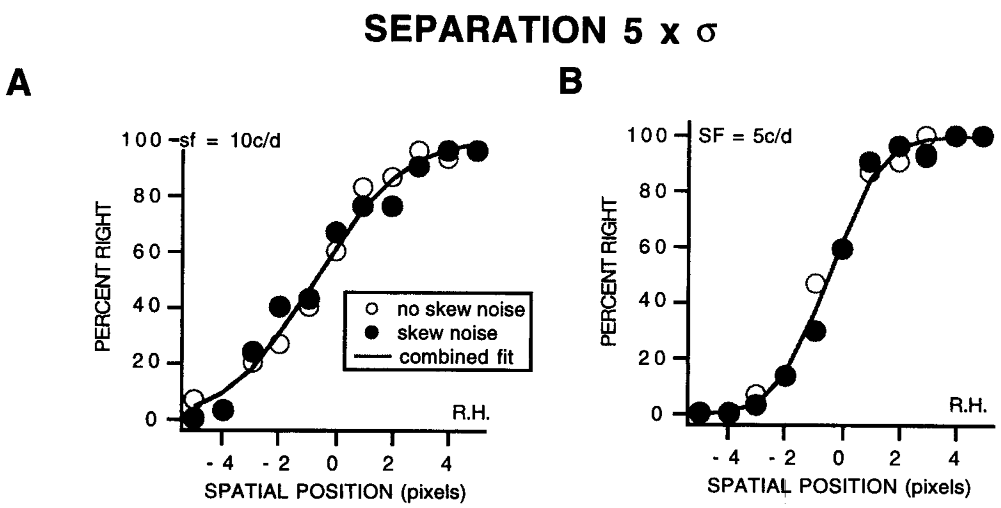

FIGURE 3. Psychometric data for the centroid-position condition are compared for the case of no skew noise (open symbols) and maximal skew noise (filled symbols). The stimuli are spatial Gabors. Results are shown for two different spatial frequencies (5 and 10 c/deg) and for two reference separations. The solid curve is the best fitting solution to the combined data for Eq. (2). See Table 1 for statistics.

from a uniform distribution of asymmetric luminance distributions. The asymmetry of any one of these stimuli was produced by using different standard deviations for the right and left sides of the horizontal Gaussian profile. This was achieved by adding a constant (skew) to, for example, the left standard deviation of the luminance distribution and subtracting it from the right standard deviation. The distribution as a whole contained 11 values extending from skew, so that at the extremes the luminance distributions were asymmetric to the same degree but mirror images of one another. Thus the 11 values of σ_L/σ_R extended from 84 min/28 min (skew noise = 1) at one extreme through 5.6 min/5.6 min (skew noise = 0) to 28 min/84 min (skew noise = 1) at the other extreme. In the final experiment (Fig. 4) we used skew noise = 1.5 and separately analysed responses at each skew. Each such run contained 1200 trials. Skew noise refers to the range of the flat distribution from which the asymmetric luminance distributions were chosen from trial to trial. In one case, all stimuli, regardless of their particular skew, were positioned with respect to the nonskewed reference stimuli by their peaks. In this case, the use of random skews resulted in centroid noise (the centroid of these distributions is given by $\sqrt{\frac{2}{\pi}} \cdot (\sigma_R - \sigma_L)$. In the other case when stimuli were positioned by virtue of their centroids, the random skew of stimuli resulted in peak noise. These conditions were run separately.

RESULTS AND DISCUSSION

The subsequent experiments involved measuring alignment accuracy for a three element alignment task in which the outer two reference stimuli had symmetric Gaussian envelopes and the middle element had an asymmetric Gaussian envelope in the x-direction. The middle element (and the two reference stimuli) was briefly displayed (temporal σ = 200 msec) in one of eleven positions straddling the aligned position and the subject was forced to indicate whether it was to the left or right of alignment. The testing grid was chosen to span the psychometric function (110 trials). The degree of luminance asymmetry of the middle element was

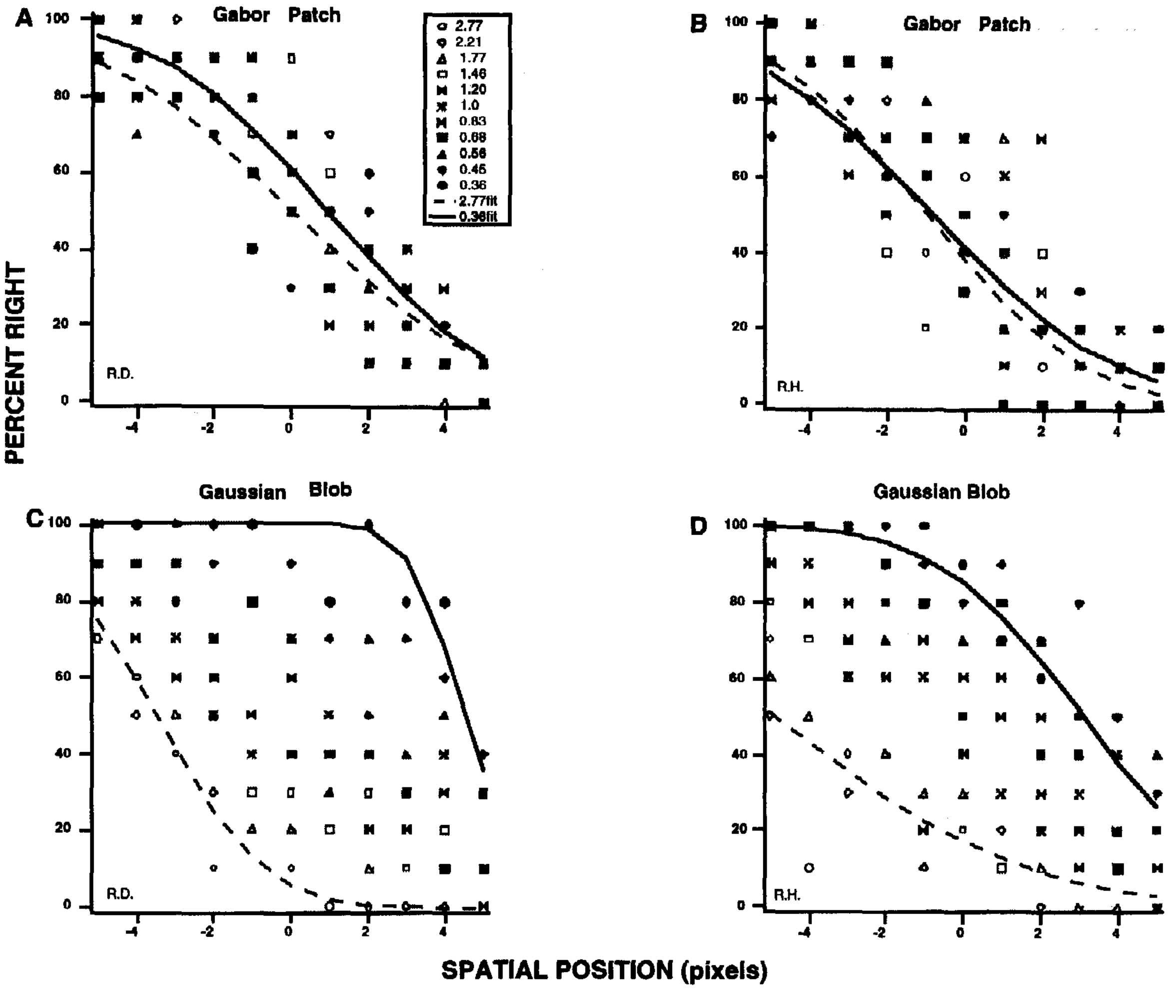

FIGURE 4.

randomized within limits (skew noise) from trial to trial and the rule concerning how these stimuli were aligned on this testing grid was one of two types. In Fig. 1, results are shown for these two conditions, one in which the asymmetric Gaussians were positioned using their peaks (filled symbols) and another where they were positioned using their centroids. Alignment error in pixels is plotted against the skew noise which represents the randomly selected degree of asymmetry of the middle element on each trial. Results are shown for two subjects, each at two separations of the reference elements. The fact that the skew was stochastic and that the stimuli were positioned about one or other cue meant that cognitive factors would be much reduced compared with the alignment of stimuli presented with stable but skewed luminance distributions. Thus if one cue or the other underlies the visual computation of stimulus location, then in this case the location of the stimuli will be fixed (and hence normal positional accuracy) whereas the appearance of the stimuli will be variable (i.e. their skewed distributions).

In all cases, performance progressively worsens when the middle element is positioned about its peak (and the centroid is allowed to vary randomly) as compared with when it is positioned about its centroid (and the peak is allowed to vary randomly). As skew noise increases, the average degree of asymmetry also increases and as a consequence so too will the difference between the peak of these asymmetric functions and their centroid. These results suggest that it is the centroid by which these functions are aligned by the human visual system.

There is a suggestion, though not statistically significant, in these data that performance may deteriorate (positive slope) even when the stimuli are positioned on the testing grid using their centroids. In the final experiment we investigate this further. We compare, within one interlaced run, no skew noise with maximal skew noise (unity in these experiments). These results are displayed in Fig. 2 for two subjects, each at two separations of the reference stimuli. The results are well fit by a single psychometric function indicating that performance does not vary as a function of skew noise when the stimuli are positioned using the centroid of the luminance profile (see Table 1 which gives the χ^2 values for single and dual fitting models to this data).

To ensure that we were not missing anything by averaging across skew polarity in the first experiment, we repeated our first experiment but kept the individual results for all 11 skews separate. All stimuli were positioned on the same grid by their centroids. There is a simple prediction, namely that if the visual system is using this feature then the results for all the different skews should fall along the *same* psychometric function. Any systematic shift in the centring of the psychometric function as a function of skew would allow rejection of the centroid hypothesis. The results (for gabor patches of 10 c/d (A&B) and Gaussian blobs (C&D)) are shown in Fig. 4 for two subjects, one (RD) naive to the aim of the experiment. Three other subjects were also run (data not displayed) with similar results. The solid and dashed curves are the best fitting error functions to the two equal but opposite *extreme* skews.

The results for the gabors support the centroid hypothesis, whereas those for the Gaussian blobs clearly do not. The results for the Gaussian blobs are consistent with a measure closer to the midpoint of the visible patch. The Gaussian blobs, unlike the gabors, allow the participation of filters tuned to much lower frequencies from which an overall shape cue for the 3-element stimulus as a whole could be derived (bowed to left/bowed to right). This is consistent with how the subjects thought they were doing the task and suggests that thinking in terms of a hardwired centroid measure for each individual element may be naive. This is at odds with the conclusions of McGraw *et al.* (1995), however, the arrangement of their stimulus with its *skewed* reference elements of *opposite* polarity would cancel out any overall shape cue produced by the lateral displacement of the central element.

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